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On the Performance Analysis of Multihop Packet Radio Systems: Part IV - Fully Connected Configurations Employing CSMA

I. Introduction

Carrier Sense Multiple Access (CSMA) [1,2,3,4] has become a well known technique, which enables efficient sharing of a data communication channel by a large population of bursty users in a ground radio environment; this environment is characterized by a propagation delay between the devices which is very small compared to the transmission time of the packet (typically 1%). Briefly, CSMA reduces the level of interference (caused by overlapping packets) in the random multiaccess environment by allowing terminals to sense the carrier due to other users' transmissions; based on this channel state information (busy or idle), the terminal takes an action prescribed by the particular CSMA protocol being used. In particular, a terminal never transmits when it senses that the channel is busy. In [1] we described and analyzed two protocols referred to as nonpersistent and p-persistent CSMA; the performance of these was given in terms of channel capacity and throughput-delay tradeoffs. It was shown that, for a ground radio single-hop environment in which all devices are within range and in line-of-sight of each other, CSMA offers a performance much superior to the ALOHA schemes. For a propagation delay equal to 1% of the transmission time of a packet, the nonpersistent protocol, for example, offers a channel capacity of over 80%, while we can only attain 36.8% for slotted ALOHA and 18.4% for pure ALOHA; packet delay is also significantly improved with CSMA. Now the questions of interest are: How does CSMA perform

in multi-hop environments? How does it compare to the ALOHA modes? Do we still enjoy the improvement in performance over ALOHA experienced in single-hop environments?

The performance analysis of multi-access schemes in multi-hop environments is a difficult task; there is no simple approach yet to tackle the general problem, nor is there a way to translate the results obtained in single-hop systems to the more complex ones. For example, we are already certain that in multi-hop environments devices are not all within range and in line-of-sight of each other; this means that on one hand the hidden terminal problem prevails and, as shown in [2], this can significantly degrade the performance of CSMA; on the other hand, not all transmissions will affect the successful reception of a packet at an immediate destination.

In parts II and III of this series [5,6] we gave an analysis of slotted ALOHA in simple multihop environments, namely the star configuration and the fully-connected configuration; in these, packets originate at terminals, are destined to a central station, and require for their transport a successful repetition by repeaters which are in line-of-sight and within range of the station. In the fully-connected configuration, all repeaters are also within range and in line-of-sight of each other; a terminal however is within range of a number of terminals, but of only one repeater. Only inbound traffic was considered. We derived the system performance in terms of capacity and throughput-delay tradeoffs and determined its dependence on various system parameters such as the transmission protocol, the network configuration, and the repeater's storage capacity.

In this note we attempt to answer the above questions by considering the simple network configurations utilized in the context of slotted ALOHA;

in particular we restrict ourselves to the fully connected one (to be described in detail below) in order to gain much of the advantage of CSMA. Basically, we shall derive the fundamental throughput-delay performance, and thus give a comparison between this and the results obtained with slotted ALOHA. Only the nonpersistent CSMA protocol will be considered because of its relative simplicity in analysis and implementation, as well as its relatively high efficiency.

To render the analysis more tractable, a number of simplifying assumptions are made. These assumptions will be introduced as needed; and to the largest extent possible we will either justify them or evaluate the effect they have on the results.

II. The Nonpersistent CSMA Revisited* [1,3]

For the sake of clarity and completeness, we repeat here the description of the nonpersistent CSMA protocol as well as the infinite population model, along with its underlying assumptions, which was previously used in analyzing the scheme. We also give the throughput analysis, a report on the determination of packet delay and a discussion of simulation results. Moreover a simple approximate model is given which characterizes the traffic of successful packets from the infinite population.

2.1 The Nonpersistent CSMA Transmission Protocol [1]

A terminal with a packet ready for transmission senses the channel and operates as follows.

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*This section consists basically of extracts of [1] and [3] which consitute the preliminaries and background necessary for the analysis presented in this note. The reader familiar with the results of [1] and [3] may skip this section perhaps with the exception of §2.5.

1) If the channel is sensed idle, it transmits the packet.

2) If the channel is sensed busy, then the terminal schedules the retransmission of the packet to some later time according to a retransmission delay distribution. At this new point in time, it senses the channel and repeats the algorithm described.

A slotted version of this can be considered in which the time axis is slotted and the slot size is τ seconds (the propagation among pairs of devices is assumed to be the same [1]). All terminals are synchronized and are forced to start transmission only at the beginning of a slot. When a packet's arrival occurs during a slot, the terminal senses the channel at the beginning of the next slot and operates according to the protocol described above.

2.2 The Infinite Population Model [1]

Assume that our traffic source consists of an infinite number of users who collectively form an independent Poisson source with an aggregate mean packet generation rate of λ packets per packet transmission time T*. This is an approximation to a large but finite population in which each user generates packets infrequently and each packet can be successfully transmitted in a time interval much less than the average time between successive packets generated by a given user. Each user in the infinite population is assumed to have at most one packet requiring transmission at any time (including any previously blocked packet). Under equilibrium conditions, λ is also the channel throughput. Because of packet interference, the achievable throughput will always be less than 1. The traffic offered to the channel from our collection of users consists not only of new packets but also of previously

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*We assume that each packet is of constant length requiring T seconds for transmission.

collided packets: this increases the mean offered traffic rate which we denote by G (packets per transmission time T) where $G \geq \lambda$. To avoid repeated conflicts, each user delays the transmission of a previously collided packet by some random time whose mean is $\bar{X}$ (chosen, for example, uniformly between 0 and $X_{max} = 2\bar{X}$).

Our two further assumptions are the following.

Assumption 1: The average retransmission delay $\bar{X}$ is large compared to T.

Assumption 2: The interarrival times of the point process defined by the start times of all the packets plus retransmissions (and reschedulings) are independent and exponentially distributed.

It is clear that Assumption 2 is violated. (We have introduced it for analytic simplicity.) However some simulation results are discussed below which show that performance results based on this assumption are excellent approximations. Moreover, in the context of slotted ALOHA it was analytically shown, in the limit as $\bar{X} \to \infty$, that Assumption 2 is satisfied.

So far, we have introduced the following notation: λ (throughput), G (offered channel traffic rate), T (packet transmission time), $\bar{X}$ (average retransmission delay) and τ (propagation delay). If we use T = 1 (for normalization), then we express τ as $a = \tau/T$ and $\bar{X}$ as $\delta = \bar{X}/T$.

2.3 Throughput Analysis [1]

Here we solve for λ in terms of G and a. The channel capacity is found by maximizing λ with respect to G. λ/G is the probability of a successful transmission or scheduling. G/λ is the average number of times a packet must be transmitted (or scheduled) until success.

Given the assumptions introduced above the (λ, G) relationship for the (unslotted) nonpersistent CSMA is given by

$$\lambda = \frac{Ge^{-aG}}{G(1+2a) + e^{-aG}} \qquad (1)$$

and for the slotted nonpersistent CSMA by

$$\lambda = \frac{aGe^{-aG}}{1 + a - e^{-aG}} \qquad (2)$$

Proof (slotted case):

We consider the time axis and we define a transmission period (TP) to be the period of time required for transmission and reception of a packet and its (possible) overlapping packets. Thus we observe on the time axis transmission periods separated by idle periods, as depicted in Fig. 1. The length of a TP is 1 + a. A TP is successful if only one packet is transmitted; the probability of this occurring is

$$P_s = \frac{aGe^{-aG}}{1 - e^{-aG}} \qquad (3)$$

Due to the memoryless property of the Poisson process, the average idle period (normalized to T) is simply

$$\bar{I} = \frac{ae^{-aG}}{1 - e^{-aG}} \qquad (4)$$

Using renewal theory arguments, the average channel utilization is given by

$$\lambda = \frac{P_s}{\bar{I} + 1 + a} \qquad (5)$$

Substituting for P_s and $\bar{I}$ the expressions found above, we get Eq (2).

Q.E.D.

Curve A in Fig. 2 is a plot of λ versus G for the slotted nonpersistent CSMA when a = 0.01.

2.4 Discussion and Delay Analysis [1,3]

The above analysis is based on renewal theory and probabilistic arguments requiring independence of random variables provided by Assumption 2. Moreover steady state conditions are assumed to exist. However from the (λ,G) relationship derived above one can see that steady state may not exist because of inherent instability of these random-access techniques. This instability is simply explained by the fact that when statistical fluctuations in G increase the level of mutual interference among transmissions, then the positive feedback causes the throughput to decrease to 0. Extensive simulation runs performed on a slotted ALOHA channel with an infinite population [7], (a system known to exhibit a behavior similar to CSMA's), have indeed shown that the assumption of channel equilibrium is not valid; in fact, after some finite time period of quasi-stationarity conditions, the channel will drift into saturation with probability one. Nevertheless, the result above is useful for, as also supported by simulation, it is meaningful for this quasi-stationary finite (and possibly long) period of time.

In the simulation model considered [1] we relax Assumptions 1 and 2 concerning the retransmission delay and the independence of arrivals for the offered channel traffic. That is, only the newly generated packets are derived independently from a Poisson distribution; collisions and uniformly distributed random retransmissions are accounted for without further assumptions. In general, our simulation results indicate the following.

1) For each value of the input rate λ, there is a minimum value δ_0 for the average retransmission delay variable such that below that value it is

impossible to achieve a throughput equal to the input rate. The higher λ is, the larger δ_o must be to prevent a constantly increasing backlog, i.e., to prevent the channel from saturating.

2) Recall that the throughput equations were based on the assumption that $\overline{X}/T = \delta >> 1$. Simulation shows that for finite values of δ, $\delta > \delta_o$, but not too large compared to 1, the system already "reaches" the asymptotic results ($\delta \to \infty$). That is, for some finite values of δ, Assumption 2 is excellent and delays are acceptable. Moreover, the comparison of the (λ,G) relationship as obtained from simulation and the results obtained from the analytic model exhibits an excellent match. Simulation experiments were also conducted to find the optimal delay; that is, the value of $\delta(S)$ which allows one to achieve the indicated throughput with the minimum delay. In Fig. 3 we plot in dashed line this delay for the nonpersistent CSMA. The values of δ used in obtaining this curve are:

$$\delta = \begin{cases} 1/2 & 0 \leq \lambda \leq 0.55 \\ 2 & 0.55 < \lambda \leq 0.65 \\ 4 & 0.65 < \lambda \leq 0.75 \end{cases}$$

In a subsequent paper [3], we formulated a Markovian model for the nonpersistent CSMA which allowed us to obtain analytically, in the case of a finite population, the throughput-delay performance as well as the effect of the retransmission delay and of the population size on the performance. With finite populations, the Markov chain in question is irreducible and ergodic and steady state is always reached; however, if the retransmission delay is not sufficiently large, then the stationary performance attained

is significantly degraded (low throughput, very high delay), such that, for all practical purposes, the channel is said to have failed; it is then called an unstable channel. With an infinite population, the Markov chain is not ergodic and stationary conditions do not exist; the channel is always unstable. For unstable channels, a stability measure is defined which consists of the average time it takes the system, starting from an empty state, to reach a state determined to be critical*[3,8]. In fact, this critical state partitions the state space into two regions, a safe region and an unsafe region. The stability measure is the average first exit time (FET) into the unsafe region. As long as the system operates in the safe region, the channel performance is acceptable; but then, of course, it is only valid over an average finite period equal to FET [3]. In Fig. 3 we also plot the throughput-delay performance for the infinite population case with guaranteed FET of 1 minute, 1 hour, and 1 month. We note that the quasi-stationary results obtained by simulation constitute a rather conservative prediction of the "true" performance of CSMA channels for a period of time of at least 1 month.

2.5 A Model for Successful Traffic

It is often the case, in multihop environments, that the output of a population of terminals, defined as the process of successful packets on the channel, constitutes the input to some other system component, say a repeater. It is therefore useful to have a simple characterization of this process. In reference [9] we have approximated the output of a nonpersistent CSMA channel by a Poisson process with a mean of λ packets per T sec. The goodness of the approximation was verified by comparing the density function of the exponential

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*In this model, the state is defined as the number of terminals with a non-empty packet buffer.

distribution to histograms of interdeparture times (i.e., time between the end of transmission of successive successful packets) obtained from simulation. Examples are shown in Fig. 4. We note that except for interdepartures in the range of one or two packet transmission times, the match is acceptable and that the samller the throughput is, the more valid is the assumption. The main criticism we have here, however, is that this approximation does not explicitly account, in the evaluation of interdeparture times, for the transmission time of the packet; the latter, as can be seen in the sequel, is an important factor in the analysis of multihop systems. Thus, we much prefer to use the following model. Let Y denote an arbitrary interdeparture time. We approximate Y by 1 + Z where Z is exponentially distributed with mean $1/\lambda - 1 = 1/\lambda'$. Thus we use

$$\Pr\{Y \leq y\} = 1 - e^{-\lambda'(y-1)} \qquad y \geq 1$$

The density function for this distribution is plotted in dashed lines in Fig. 4. The comparison with the interdeparture histograms shows that this approximation is very acceptable as well.

III. Analysis of Fully-Connected Configurations

3.1 System Configuration and Transmission Protocol

We consider in this note the fully-connected two-hop network configuration [6] in which all repeaters are within range and in line-of-sight of each other and of the station. With each repeater is associated an infinite population of terminals generating traffic which is destined to the station (see Fig. 5). Each repeater is provided with a finite storage capacity which can accomodate exactly one packet. Termina' follow the nonpersistent protocol described earlier in section 2.1. Repeaters also use the nonpersistent CSMA

mode as described in the following. A repeater which has completed the successful reception of a packet from its associated population of terminals, transmits the packet without delay. The repeater is guaranteed that the channel is idle at the end of a correct reception since, given the system connectivity, all repeaters must have been quiet during the entire reception time of the packet. (This corresponds to the so-called immediate-first-transmission (IFT) protocol considered in [6] for the slotted ALOHA mode as well.) This first transmission of the packet by the repeater may be unsuccessful due to collisions with transmissions from other active repeaters. The rescheduling of the packet is assumed to be geometrically distributed: the unsuccessful repeater resenses the channel in the current slot with a fixed probability ν; of course a retransmission will result only if the channel is sensed idle. Just as in [5,6] we shall assume that devices learn about their success or failure instantaneously at the end of their transmission period. We define the network throughput S to be the average number of packets received at the station per T seconds. The packet delay D is defined to be the time since the packet originates at the terminal until it is successfully received at the station; as in [5,6], we distinguish the two components: D_a, the access delay and D_n the network delay. We are seeking here the throughput-delay performance of such CSMA networks. N denotes the number of repeaters present in the system.

3.2 The Single-Repeater Case

Before we proceed with the general model and its underlying assumptions, we consider here the simple case N = 1. It is clear that in this case repeater's transmissions are successful. Considering the infinite population model described in section 2.2, the analysis of this system differs from the

single-hop one given in section 2.3 by the simple fact that a successful transmission here requires 2T seconds instead of only T. The probability of a successful transmission P_s and the average idle period $\bar{I}$ are as expressed in Eqs (3) and (4) respectively; the average busy period however is given by

$$\bar{B} = (1 - P_s)(1 + a) + P_s(2 + 2a) \tag{7}$$

The network throughput is then expressed as

$$S = \frac{P_s}{\bar{I} + \bar{B}} = \frac{aGe^{-aG}}{1 + a - e^{-aG} + aGe^{-aG}(1 + a)} \tag{8}$$

Curve B in Fig. 2 is a plot of S versus G for this simple case. We note here that the network capacity is about 46% of the total available bandwidth while, in single hop environments, we were able to achieve 85%; the channel utilization at the repeater, however, is 92%! This example shows that the introductio of a repeater, and thus an extra hop for each packet transmission, has significantly decreased the net throughput of a centralized network. Next, we consider the more general case N > 1.

3.3 Model Assumptions

Consider for each population of terminals T_i a time line which exhibits packet transmissions from T_i only. Consider also a time line R which exhibits packet transmissions from repeaters only. On each such time line we observe an alternate sequence of transmission and idle periods. The processes defining these time lines are evidently dependent on each other in a rather complex way; the dependence is determined by the particular system connectivity. To render the analysis tractable, we introduce here the following two assumptions.

Assumption 3. When sensing the channel, a repeater R_i can distinguish between the presence of carrier due to other repeaters and carrier due to transmissions by its associated population of terminals T_i*.

The simplification gained here lies in the fact that the decision made by a repeater to transmit its packet is solely dependent on the state of the repeaters population. The performance evaluation will be rather slightly optimistic; indeed, delaying transmission by a repeater because of terminals' transmission (whether successful or not**) would only degrade the system performance. It is to be noted, however, that the effect of this assumption will be smaller as N gets larger. Our next assumption is

Assumption 4. Similarly, terminals do not inhibit transmission when their associated repeater is transmitting.

It is clear that this assumption is violated in the environment under consideration. By introducing it, we simplify the problem in that we allow the processes defining each time line T_i to be independent of the repeaters time line; the successful transport of a packet from T_i to R_i, on the other hand, will be considered dependent on the state of R, as will be seen in the analysis below. The effect of Assumption 4 is to provide a pessimistic evaluation of system performance; indeed transmissions from T_i which start during a transmission period of R_i are useless and contribute to a higher traffic rate on time line T_i. However, it is to be noted here again that this effect

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*The distinction between the two different carriers can be made possible in a real system if the discrepancy in power levels is significant (the repeater transmit power being for example much greater than terminals' transmit power) or if different data rates are used.

**Repeaters possess single packet buffers.

will be smaller as N gets larger. For N = 10, for example, T_i can only hear 10% of the repeaters' traffic! (In section 3.6 below, we evaluate this effect for the N = 1 case, which was analyzed exactly in 3.2 above.)

With the above two assumptions, the correlation among the time lines is reduced; Fig. 6 exhibits instances of these time lines. The analysis follows.

3.4 Analysis

Similarly to the analysis given in Parts II and III [5,6] for slotted ALOHA, we shall treat here the inner-hop and the terminal-to-repeater hop separately.

Consider time line R on which we observe an alternate sequence of transmission periods and idle periods. As in [3], we consider the imbedded slots defined to be the first slot of each idle period (see Fig. 7). The intervals between two consecutive imbedded slots are defined as cycles. Let n^{t_e} denote the number of active repeaters* in slot t_e. We show that n^{t_e} is a Markov chain and determine its transition probabilities.

Given $n^{t_e} = n$, let I_n denote the length of the idle period (in slots). An idle period ends in a slot if either an active repeater decides to start transmission in that slot or a successful transmission to a passive repeater from its associated population of terminals is completed in that slot (since the repeater immediately relays it) or both. A transmission from T_i is said to be T_i-successful if it is free of collision from other terminals in T_i. It is clear that for a T_i-successful transmission to be successfully received at repeater R_i (considered inactive), this transmission should entirely take place during an idle period of time line R. Consider the imbedded slot t_e

*An active repeater is a repeater with a non-empty buffer.

and assume $n^{t_e} = n$. Let then J_n denote the time until some active repeater decides to sense the channel (and hence to transmit if the channel is idle). J_n is geometrically distributed; its density function is given by

$$\Pr\{J_n = k \text{ slots}\} = (1 - \nu)^{n(k-1)}[1 - (1 - \nu)^n] \qquad (9)$$

With $n^{t_e} = n$, there are N-n inactive repeaters. Let R_{i_j} again denote one of them. By the independence assumptions we regard time line T_{i_j} to be entirely governed by the processes of the infinite population model, where the rate of T_{i_j}-successful packets is λ and the total rate of sense points is G. By the same assumptions, we can reasonably assume that, relative to time line T_{i_j}, the end of a cycle represents a random look in time; accordingly, the probability that this point falls in a transmission period of T_{i_j} is $1 - \lambda/G$, and in an idle period, λ/G; these two instances are shown in Fig. 8. We let Y_{i_j} denote the time since $_e$ until the end of the transmission period; its distribution is then given by

$$\Pr\{Y_{i_j} \leq y\} = \frac{\lambda}{G} + (1 - \frac{\lambda}{G})\frac{y}{T} \qquad 0 \leq y \leq T \qquad (10)$$

Given that a transmission from T_{i_j} requires T slots*, it is clear that no successful reception at repeater R_i can take place <u>before</u> slot $t_e + Y_{i_j} + T \triangleq t_e + Y'_{i_j}$. From the characterization of successful traffic introduced in section 2.5 above, we note that, following $t_e + Y'_{i_j}$ the arrival process from T_{i_j} to R_{i_j} can be represented by a Bernoulli process, whereby the probability

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*Consider the slot in this section to represent the time unit; T then represents the number of slots per packet transmission time and a = 1/T; the performance measures however will still be given normalized to T.

of completion of a correct reception in a slot is $a\lambda'$, with $\lambda' = 1/(1/\lambda - 1)$. Without loss of generality, we let $0 \triangleq Y'_{i_0} \leq Y'_{i_1} \leq \ldots \leq Y'_{i_{N-n}} < \infty$. It is clear that for any slot t, $t_e + Y'_{i_j} \leq t < t_e + Y'_{i_{j+1}}$, (and under the condition that no arrival took place to any inactive repeater prior to t,) the arrival process in slot t is binomial such that

$$\Pr\{k \text{ packet receptions completed in } t,\ 0 \leq k \leq j\} = \binom{j}{k}(a\lambda')^k(1-a\lambda')^{j-k} \tag{11}$$

To avoid the great complexity involved in treating the problem exactly, we choose here to derive an upper and lower bound on performance by considering much simpler arrival processes. Let $Y'_{min} = Y'_{i_1}$ and $Y'_{max} = Y'_{i_{N-n}}$ The upper bound is obtained by considering the following:

$$\Pr\{k \text{ packet receptions completed in } t,\ 0 \leq k \leq N - n\}$$

$$= \begin{cases} 0 & t < t_e + Y'_{min} \\ \binom{N-n}{k}(a\lambda')^k\ (1-a\lambda')^{N-n-k} & t_e + Y'_{min} \leq t < \infty \end{cases} \tag{12}$$

The lower bound is obtained by substituting Y'_{max} for Y'_{min} in Eq. (12). Let Y'_m denote interchangeably Y'_{min} and Y'_{max} The subscript m will be replaced by min or max where needed. If $J_n < Y'_m$ then the idle period ends because of the start of a transmission from an active repeater; if $J_n \geq Y'_m$ then arrivals to passive repeaters are possible, and for each slot thereon it is the contention of both active repeaters and passive repeaters just completing reception that determine the end of the idle period in that slot. The system state does not vary over a transmission period of time line R.

With these considerations, the transition probabilities between consecutive imbedded points are simply given by

$$P_{0,j} = \begin{cases} \dfrac{N(a\lambda')(1-a\lambda')^{N-1}}{1-(1-a\lambda')^{N}} & j = 0 \\ 0 & j = 1 \\ \dfrac{\binom{N}{k}(a\lambda')^{k}(1-a\lambda')^{N-k}}{1-(1-a\lambda')^{N}} & j > 1 \end{cases} \tag{13}$$

$$P_{N,j} = \begin{cases} 0 & j < N-1 \\ \dfrac{N\nu(1-\nu)^{N-1}}{1-(1-\nu)^{N}} & j = N-1 \\ 1 - \dfrac{N\nu(1-\nu)^{N-1}}{1-(1-\nu)^{N}} & j = N \end{cases} \tag{14}$$

and for $1 \leq n \leq N-1$

$$P_{n,j} = \begin{cases} 0 & j < n-1 \\ \Pr\{J_n < Y'_m\} \dfrac{n\nu(1-\nu)^{n-1}}{1-(1-\nu)^{n}} + \Pr\{J_n \geq Y'_m\} \dfrac{(1-a\lambda')^{N-n} n\nu(1-\nu)^{n-1}}{1-(1-\nu)^{n}(1-a\lambda')^{N-n}} & j = n-1 \\ \Pr\{J_n < Y'_m\} \dfrac{1-n\nu(1-\nu)^{n-1}-(1-\nu)^{n}}{1-(1-\nu)^{n}} & \\ \quad + \Pr\{J_n \geq Y'_m\} \dfrac{(1-a\lambda')^{N-n}[1-n\nu(1-\nu)^{n-1}-(1-\nu)^{n}]+(N-n)a\lambda'(1-a\lambda')^{N-n-1}(1-\nu)^{n}}{1-(1-\nu)^{n}(1-a\lambda')^{N-n}} & j = n \\ \Pr\{J_n \geq Y'_m\} \dfrac{(N-n)a\lambda'(1-a\lambda')^{N-n-1}[1-(1-\nu)^{n}]}{1-(1-\nu)^{n}(1-a\lambda')^{N-n}} & j = n+1 \\ \Pr\{J_n \geq Y'_m\} \dfrac{\binom{N-n}{j-n}(a\lambda')^{j-n}(1-a\lambda')^{N-j}}{1-(1-\nu)^{n}(1-a\lambda')^{N-n}} & j > n+1 \end{cases} \tag{15}$$

We now derive the expressions for $Pr\{J_n \geq Y'_{min}\}$ and $Pr\{J_n \geq Y'_{max}\}$. Given $n^{t_e} = n$, and the distribution for Y_{i_j} given in Eq. (10), we have

$$Pr\{Y'_{max} \leq T + y\} = \left[\frac{\lambda}{G} + (1 - \frac{\lambda}{G})\frac{y}{T}\right]^{N-n} \qquad 0 \leq y \leq T \qquad (16)$$

$$Pr\{Y'_{min} > T + y\} = \left[(1 - \frac{\lambda}{G})(1 - \frac{y}{T})\right]^{N-n} \qquad 0 \leq y \leq T \qquad (17)$$

From the distribution of J_n given in Eq. (9) we note that

$$Pr\{J_n \geq k\} = (1 - \nu)^{n(k-1)} \qquad (18)$$

Using Eqs. (16) through (18) we have

$$Pr\{J_n \geq Y'_{max}\} = (1-\nu)^{n(T-1)}\left[\left(\frac{\lambda}{G}\right)^{N-n} + \int_{y=0}^{T}(1-\nu)^{ny}\, dPr\{Y'_{max} \leq T+y\}\right] \qquad (19)$$

$$Pr\{J_n \geq Y'_{min}\} = (1-\nu)^{n(T-1)}\left[1-\left(1-\frac{\lambda}{G}\right)^{N-n} + \int_{y=0}^{T}(1-\nu)^{ny}\, dPr\{Y'_{min} \leq T+y\}\right] \qquad (20)$$

It is shown in Appendix A, that the solution of the above integrals yields the following expressions:

$$\begin{aligned} Pr\{J_n \geq Y'_{max}\} &= (1-\nu)^{n(T-1)}\left(\frac{\lambda}{G}\right)^{N-n} \\ &+ (1-\nu)^{n(2T-1)}(N-n)\left[\frac{1}{\alpha} + \sum_{k=1}^{N-n-1}(-1)^k \frac{(N-n-1)!}{(N-n-1-k)!}\frac{1}{\alpha^{k+1}}\right] \\ &- (1-\nu)^{n(T-1)}(N-n)\left[\frac{(\lambda/G)^{N-n-1}}{\alpha} + \sum_{k=1}^{N-n-1}(-1)^k \frac{(N-n-1)!}{(N-n-1-k)!}\frac{(\lambda/G)^{N-n-1-k}}{\alpha^{k+1}}\right] \end{aligned} \qquad (21)$$

$$\Pr\{J_n \geq Y'_{min}\} = (1-\nu)^{n(T-1)}\left[1-(1-\frac{\lambda}{G})^{N-n}\right]$$

$$+ (1-\nu)^{n(T-1)}(N-n)\left[(1-\nu)^{nT}\frac{(N-n-1)!}{\alpha^{N-n}} - \frac{(1-\lambda/G)^{N-n-1}}{\alpha}\right.$$

$$\left. - \sum_{k=1}^{N-n-1}\frac{(N-n-1)!}{(N-n-1-k)!}\frac{(1-\lambda/G)^{N-n-1-k}}{\alpha^{k+1}}\right] \qquad (22)$$

where $\alpha = -\dfrac{T\,\mathrm{Log}[(1-\nu)^n]}{(1-\frac{\lambda}{G})}$

Thus we have so far determined that n^{t_e} is a Markov chain and derived its transition probability matrix, which we denote here by P. The chain is irriducible and ergodic and a stationary distribution $\Pi = \{\pi_0, \pi_1, \ldots, \pi_N\}$ exists where

$$\pi_i = \lim_{t_e \to \infty} \Pr\{n^{t_e} = i\}$$

Π is obtained by solving recursively the system $\Pi = \Pi P$. Now, we proceed with the derivation of the performance measures, namely the network throughput S and the network delay D_n. We have defined a cycle to be the interval of time separating two successive imbedded slots; a cycle consists of an idle period followed by a transmission period. Given that $n^{t_e} = n$, let I_n denote the length of the idle period; the transmission period is of length T + I; the cycle length is $I_n + T + 1$. The probability of a successful transmission by the repeaters over the cycle, which we denote by S_n is expressed as

$$S_n = \Pr\{J_n < Y'_m\}\,\frac{n\nu(1-\nu)^{n-1}}{1-(1-\nu)^n}$$

$$+ \Pr\{J_n \geq Y'_m\}\,\frac{n\nu(1-\nu)^{n-1}(1-a\lambda')^{N-n} + (N-n)a\lambda'(1-a\lambda')^{N-n-1}(1-\nu)^n}{1-(1-\nu)^n(1-a\lambda')^{N-n}} \qquad (23)$$

In this expression, we distinguished the case $J_n < Y'_m$ where only active repeaters contend on the channel, and the case $J_n \geq Y'_m$ where arrivals may contend as well. Let σ_n denote the average sum of active repeaters over all slots in the cycle. $\bar{I}_n$ denoting the average length of I_n, it is expressed as

$$\sigma_n = (\bar{I}_n + T + 1)n + (T + 1)\ \Pr\{J_n \geq Y'_m\}\ \frac{(N-n)\ a\lambda'}{1-(1-\nu)^n\ (1-a\lambda')^{N-n}} \qquad (24)$$

By renewal theory arguments, we write the stationary system throughput S and the stationary average number of active repeaters $\bar{n}$ respectively as

$$S = \frac{\sum_{n=0}^{N} \pi_n\ S_n\ T}{\sum_{n=0}^{N} \pi_n(\bar{I}_n + T + 1)} \qquad (25)$$

$$\bar{n} = \frac{\sum_{n=0}^{N} \pi_n\ \sigma_n}{\sum_{n=0}^{N} \pi_n(\bar{I}_n + T + 1)} \qquad (26)$$

By Little's result, the average network delay D_n is given by

$$D_n = \frac{\bar{n}}{S} \qquad (27)$$

We are now left with the determination of $\bar{I}_n$. Given that $Y'_m = y$, the average idle period is simply given by

$$\bar{I}_{n/Y'_m=y} = \Pr\{J_n < y\}\ \bar{I}_{n/J_n<y, Y'_m = y} + \Pr\{J_n \geq y\} \left[y + \frac{1}{1-(1-\nu)^n(1-a\lambda')^{N-n}} \right] \qquad (28)$$

Let $n \neq 0,N$ we first have

$$\Pr\{J_n < y\} = 1 - \Pr\{J_n > y - 1\} = 1 - (1-\nu)^{n(y-1)} \tag{29}$$

For $0 \leq k \leq y-1$ we have

$$\Pr\{I_n = k \;/\; Y'_m = y,\ J_n < y\} = \frac{(1-\nu)^{n(k-1)}[1-(1-\nu)^n]}{1-(1-\nu)^{n(y-1)}} \tag{30}$$

The average idle period in this case is

$$\bar{I}_{n/Y'_m = y, J_n < y} = \sum_{k=0}^{y-1} \frac{k(1-\nu)^{n(k-1)}[1-(1-\nu)^n]}{1-(1-\nu)^{n(y-1)}}$$

$$= \frac{1-(1-\nu)^{ny} - y(1-\nu)^{n(y-1)}[1-(1-\nu)^n]}{[1-(1-\nu)^n][1-(1-\nu)^{n(y-1)}]} \tag{31}$$

Thus, for $n \neq 0,N$,

$$\bar{I}_{n/Y'_m = y} = \frac{1-(1-\nu)^{ny} - y(1-\nu)^{n(y-1)}[1-(1-\nu)^n]}{1-(1-\nu)^n}$$

$$+ y(1-\nu)^{n(y-1)} + \frac{(1-\nu)^{n(y-1)}}{1-(1-\nu)^n(1-a\lambda')^{N-n}}$$

$$= \frac{1}{1-(1-\nu)^n} + (1-\nu)^{n(y-1)}\left[\frac{1}{1-(1-\nu)^n(1-a\lambda')^{N-n}} - \frac{(1-\nu)^n}{1-(1-\nu)^n}\right] \tag{32}$$

Removing the condition Y'_m, we finally have

$$\bar{I}_n = \frac{1}{1-(1-\nu)^n} + \Pr\{J_n > Y'_m\}\left[\frac{1}{1-(1-\nu)^n(1-a\lambda')^{N-n}} - \frac{(1-\nu)^n}{1-(1-\nu)^n}\right] \tag{33}$$

where Y'_m can be replaced by either Y'_{min} or Y'_{max}.

When $n = 0$, $\Pr\{J_n < y\} = 0$ and Eq. (28) is written as

$$\bar{I}_{o/Y'_m = y} = y + \frac{1}{1-(1-a\lambda')^N} \tag{34}$$

Removing the condition on Y'_m, we get for the lower bound case

$$\bar{I}_{o,max} = \frac{1}{1-(1-a\lambda')^N} + T + N(1-\frac{\lambda}{G})\frac{1}{T}\int_0^T y\left[\frac{\lambda}{G} + (1-\frac{\lambda}{G})\frac{y}{T}\right]^{N-1} dy$$

$$= \frac{1}{1-(1-a\lambda')^N} + T + N(1-\frac{\lambda}{G})\,T\left[\sum_{k=0}^{N-1}\binom{N-1}{k}\frac{(\lambda/G)^k(1-\lambda/G)^{N-1-k}}{N-1-k+2}\right] \tag{35}$$

and for the upper bound case

$$\bar{I}_{o,min} = \frac{1}{1-(1-a\lambda')^N} + T + N(1-\frac{\lambda}{G})\frac{1}{T}\int_0^T y\left[(1-\frac{\lambda}{G}) - (1-\frac{\lambda}{G})\frac{y}{T}\right]^{N-1} dy$$

$$= \frac{1}{1-(1-a\lambda')^N} + T + N(1-\frac{\lambda}{G})^N\,T\left[\sum_{k=0}^{N-1}\binom{N-1}{k}\frac{(-1)^{N-1-k}}{N-1-k+2}\right] \tag{36}$$

For the case $n = N$, we simply have

$$\bar{I}_N = \frac{1}{1-(1-\nu)^N} \tag{37}$$

This completes the inner-hop analysis.

3.5 Calculation of D_a

As with slotted ALOHA networks in Part II [5], we estimate here the access delay D_a by

$$D_a = \frac{1}{1-B}\,D_{NPCSMA}(\lambda) + \frac{B}{1-B}\,\delta(\lambda) \tag{38}$$

where $D_{NPCSMA}(\lambda)$ is the average packet delay of an infinite population

employing the nonpersistent CSMA protocol and whose output is λ; $\delta(\lambda)$ is the optimum average retransmission delay and is given in section 2.4, and B is the probability that a T_i-successful packet gets blocked at the receiving repeater and is expressed as

$$B = 1 - \frac{S}{N\lambda} \tag{39}$$

IV. Discussion and Results

4.1 Note on the Effect of Assumption 4

Consider the simple case of N = 1. We evaluate here the effect of assumption 4 by giving an approximate analysis and then compare the results to those obtained by the exact analysis given in section 3.2 above. With N = 1, the state of the system at all imbedded points is trivially $n^{t_e} = 0$. The average idle period is given by Eq. (35) or Eq. (36)

$$\bar{I}_o = \frac{1}{a\lambda\tau} + T + (1 - \frac{\lambda}{G})\frac{T}{2} = (\frac{1}{\lambda} + \frac{1-\lambda/G}{2})T \tag{40}$$

The probability of success of a transmission on time line R is 1. The network throughput, using the normalized time units, is simply

$$S = \frac{1}{\frac{1}{\lambda} + \frac{1-\lambda/G}{2} + 1 + a} \tag{41}$$

where λ and G are related through Eq. (2). We plot S versus G in Fig. 2 above as curve C. With this analysis we have been able to predict a system capacity of only 0.32 versus 0.46 obtained by the exact analysis. Thus the independence assumption in this worse case of N = 1 has provided an error of 30% in predicting the capacity. As pointed out earlier, the error is due to the excessive transmissions by the terminal population which would not

be present had we taken the proper connectivity into account; the error is represented by the term $\frac{1-\lambda/G}{2}$ which represents the additional waiting until an arrival at the repeater is successful; near capacity, this term approaches one half of a transmission time since λ/G is then small; that is, the random look falls during a transmission period with a high probability. It is not surprising to note that the removal of this term will lead to an expression for S in terms of G which is identical to the one obtained through the exact analysis, namely Eq. (8)! The effect of assumtion 4 will be much smaller for N = 2, since normally terminals in a given population will be out of range of one repeater, that is of half of the repeaters' traffic. With N = 10, the results will be even more accurate. At any rate, these results represent a pessimistic evaluation of performance.

4.2 Discussion of Numerical Results

We show in Table 1 numerical results obtained for various values of the parameters N, λ, and ν. The numerical results have shown the following.

1. The performance is not too sensitive to variations in ν; however a very small value of ν ($\nu \leq 0.001$) may induce degradation in performance.

2. The network delay is not much larger than one. The access delay, on the other hand, is the predominant factor in packet delays as the throughput increases.

We explain these points by noting that with the nonpersistent CSMA, as long as N is not too large ($N \leq 10$), the probability that a transmission is successful is very close to 1. Moreover, with the IFT protocol used here, the repeater is guaranteed that the channel is idle at the end of a correct reception since, given the system connectivity, all repeaters must have been quiet during the entire reception time of the packet. (With a network delay

as small as this, there was no need to consider other protocols than IFT.)

3. The difference between the lower bound and upper bound on S and B becomes important as λ increases. The discrepancy is even more important for larger values of N. Thus, no good estimate is obtained for S (and B). Network delay, however, is <u>not</u> affected by the approximation and an accurate estimate is obtained by this analysis.

Table 1

N	λ	ν	$(D_n)_{min}$	$(D_n)_{max}$	S_{min}	S_{max}	B_{min}	B_{max}
2	0.1	0.001	1.023	1.023	0.1534	0.1512	0.233	0.[illegible]1
		0.01	1.013	1.013	0.1535	0.1513	0.232	0.[illegible]3
		0.1	1.012	1.012	0.1535	0.1513	0.232	0.[illegible]3
		0.5	1.012	1.012	0.1535	0.1513	0.232	0.243
2	0.8	0.001	1.528	1.528	0.4009	0.3544	0.749	0.778
		0.01	1.116	1.116	0.4198	0.3645	0.737	0.772
		0.1	1.077	1.077	0.4232	0.3668	0.735	0.770
		0.5	1.092	1.092	0.4220	0.3659	0.736	0.771
5	0.1	0.001	1.072	1.072	0.2618	0.2476	0.476	0.504
		0.01	1.022	1.022	0.2623	0.2479	0.475	0.504
		0.1	1.017	1.017	0.2624	0.2481	0.475	0.504
		0.5	1.019	1.019	0.2624	0.2480	0.475	0.504
5	0.7	0.001	2.513	2.371	0.4535	0.3461	0.870	0.901
		0.01	1.270	1.252	0.4620	0.3477	0.868	0.900
		0.1	1.163	1.163	0.4685	0.3525	0.866	0.899
		0.5	1.200	1.200	0.4650	0.3505	0.867	0.900

Examining closely the intermediate numberical results, we observe that the stationary distributions Π_{min} and Π_{max} are "identical"* for the optimum ($\nu = 0.1$) the probability of success $[(P_s)_n]_{min}$ and $[(P_s)_n]_{max}$ are also very close to each other and close to 1; the average idle periods $[\overline{I}_n]_{min}$ and $[\overline{I}_n]_{max}$, on the contrary, show important differences affecting significantly the performance evaluation. To overcome this difficulty we recourse to simulation and estimate separately, $(P_s)_n$ and $\overline{I}_n$ for $n = 0,1,\ldots,N$; then using Π_{min} or Π_{max} we derive the performance measures. The simulation of the subprocesses I_n (and $(P_s)_n$) is a much simpler task than a complete simulation of the system. The details follow.

4.3 Simulation of I_n and $(P_s)_n$

Let $n^{te} = n$. The algorithm used to generate one sample of I_n, $(P_s)_n$ and σ_n is as follows.

1. Generate N-n random variables $\{Y'_{i_j}\}_{j=1}^{N-n}$ according to the distribution given in Eq. (10). Without loss of generality, we assume that

$$0 = Y'_{i_0} \leq Y'_{i_1} \leq Y'_{i_2} \leq \cdots \leq Y'_{i_{N-n}} \leq Y'_{i_{N-n+1}} = \infty$$

2. $j \leftarrow 0$

3. Generate a random variable J_n^j such that

$$\Pr\{J_n^j = k\} = \left[(1-\nu)^n (1-a\lambda')^j\right]^{(k-1)} \left[1 - (1-\nu)^n (1-a\lambda')^j\right] \qquad (42)$$

If $J_n^j < Y'_{i_{j+1}} - Y'_{i_j}$ then do;

$$I_n = Y'_{i_j} + J_n^j \qquad (43)$$

---

*Accurate within 4 decimals (the accuracy used in printing the results).

$$(P_s)_n = \frac{n\nu(1-\nu)^{n-1}(1-a\lambda')^j + j\, a\lambda'(1-a\lambda')^{j-1}(1-\nu)^n}{1 - (1-\nu)^n (1-a\lambda')^j} \tag{44}$$

$$\sigma_n = (I_n + T + 1)n + \frac{a\lambda'(T+1)}{1-(1-\nu)^n(1-a\lambda')^j} \tag{45}$$

stop;

else $j \leftarrow j + 1$; repeat this step.

Let L be the number of samples needed. The algorithm is repeated L times. The estimates for $\overline{I}_n$, $(P_s)_n$, σ_n , denoted by $(\overline{I}_n)_{sim}$, $[(P_s)_n]_{sim}$, $(\sigma_n)_{sim}$ respectively, are obtained by just taking the average over the L samples. The estimates for the performance measures S, and D_n are obtained by using Eqs. (25), (26) and (27) in which we substitute $(\overline{I}_n)_{sim}$, $[(P_s)_n]_{sim}$, $(\sigma_n)_{sim}$ for I_n , $(P_s)_n$ and σ_n respectively.

4.4 The Throughput-Delay Tradeoff; CSMA versus Slotted ALOHA

We plot in Fig. 9 the throughput-delay tradeoff for N=2, 5 and 10. We note a slight improvement as N increases. Contrary to the slotted ALOHA case [5,6] in which we noted that for $N \geq 3$, the inner-hop constitutes practically the bottleneck, with CSMA the inner-hop is extremely efficient and the terminal to repeater hop becomes more critical. As N increases, the input rate λ required at each repeater to produce a given throughput S, is smaller and therefore the "wasted" time on the time lines T_i represented by the variables Y_i' is also less important; accordingly it is possible to have a larger number of simultaneous receptions at various repeaters, and therefore to achieve a higher system capacity; the access delay D_a is also smaller with smaller λ.

As for the comparison between the performances of slotted ALOHA and CSMA networks, we summarize in Figs. 10, 11 and 12 the throughput-delay tradeoff

for all systems considered in this series of notes for N = 2, 5 and 10 respectively. We note that CSMA offers an improvement over slotted ALOHA, which becomes more significant as N increases. Fig. 13 displays the system capacity.

Appendix A

Derivation of $\Pr\{J_n \geq Y'_{max}\}$ and $\Pr\{J_n \geq Y'_{min}\}$

$$(i) \quad \Pr\{J_n \geq Y'_{max}\} = (1-\nu)^{n(T-1)}\left[\left(\frac{\lambda}{G}\right)^{N-n} + \int_{y=0}^{T} (1-\nu)^{ny}\, d\Pr\{Y'_{max} \leq T + y\}\right]$$

$$= (1-\nu)^{n(T-1)}\left[\left(\frac{\lambda}{G}\right)^{N-n}\right.$$

$$\left. + (N-n)\left(1 - \frac{\lambda}{G}\right)\frac{1}{T}\int_{0}^{T} (1-\nu)^{ny}\left[\frac{\lambda}{G} + \left(1 - \frac{\lambda}{G}\right)\frac{y}{T}\right]^{N-n-1} dy\right]$$

(A.1)

We note that the integral is of a known form

$$\int x^m \ell^{\alpha x}\, d = e^{\alpha x}\left[\frac{x^m}{\alpha} + \sum_{k=1}^{m} (-1)^k \frac{m(m-1)\ldots(m-k+1)}{\alpha^{k+1}} x^{m-k}\right] \quad (A.2)$$

Define the following:

$$\mu = (1-\nu)^n \quad (A.3)$$

$$m = N-n-1 \quad (A.4)$$

$$\beta = \lambda/G \quad (A.5)$$

$$\gamma = (1-\lambda/G)/T \quad (A.6)$$

$$x = \beta + \gamma y \quad (A.7)$$

$$\alpha = \frac{\text{Log } \mu}{\gamma} \quad (A.8)$$

then the integral in Eq. (A.1) above becomes

$$\int_0^T (1-\nu)^{ny} \left[\frac{\lambda}{G} + (1-\frac{\lambda}{G}) \frac{y}{T} \right]^{N-n-1} dy$$

$$= \frac{e^{-\alpha\beta}}{\gamma} \int_\beta^1 x^m e^{\alpha x} dx$$

$$= \frac{e^{-\alpha\beta}}{\gamma} \left\{ e^{\alpha} \left[\frac{1}{\alpha} + \sum_{k=1}^{m} (-1)^k \frac{m!}{(m-k)!} \frac{1}{\alpha^{k+1}} \right] \right.$$

$$\left. - e^{\alpha\beta} \left[\frac{\beta}{\alpha} + \sum_{k=1}^{m} (-1)^k \frac{m!}{(m-k)!} \frac{\beta^{m-k}}{\alpha^{k+1}} \right] \right\} \qquad (A.9)$$

Substituting this expression in Eq. (A.1), and replacing m, β, and γ by their expressions given in Eqs. (A.4), (A.5) and (A.6) respectively, we get Eq. (21)

$$\text{(ii)} \quad \Pr\{J_n \geq Y'_{min}\} = (1-\nu)^{n(T-1)} \left[1-(1-\frac{\lambda}{G})^n + \int_0^T (1-\nu)^{ny} \, d\Pr\{Y'_{min} \leq T + y \right.$$

$$= (1-\nu)^{n(T-1)} \left[1-(1-\frac{\lambda}{G})^n \right.$$

$$\left. + (N-n)(1-\frac{\lambda}{G}) \frac{1}{T} \int_0^T (1-\nu)^{ny} \left[1-\frac{\lambda}{G} - (1-\frac{\lambda}{G}) \frac{y}{T} \right]^{N-n-1} dy \right.$$

(A.10)

Define $\beta' = 1 - \beta$ (A.11)

$\gamma' = -\gamma$ (A.12)

$x = \beta' + \gamma' y$ (A.13)

$\alpha' = -\alpha$ (A.14)

then the integral in Eq. (A.10) above is again of the form given in Eq. (A.2)

$$\int_0^T (1-\nu)^{ny} \left[1 - \frac{'\lambda}{G} - (1-\frac{\lambda}{G})\frac{y}{T}\right]^{N-n-1} dy$$

$$= \frac{e^{-\alpha'\beta'}}{\gamma'} \int_\beta^0 x^m e^{\alpha' x} dx$$

$$= \frac{e^{-\alpha'\beta'}}{\gamma'} \left\{(-1)^m \frac{m!}{(\alpha')^{m+1}} - e^{\alpha'\beta'} \left[\frac{(\beta')^m}{\alpha'} + \sum_{k=1}^{m} (-1)^k \frac{m!}{(m-k)!} \frac{(\beta')^{m-k}}{(\alpha')^{k+1}}\right]\right\}$$

(A.15)

Substituting this for the integral in Eq. (A.10) and replacing m, β', and γ' by their expressions as defined in Eqs. (A.4), (A.11) and (A.12) respectively, we obtain Eq. (22).

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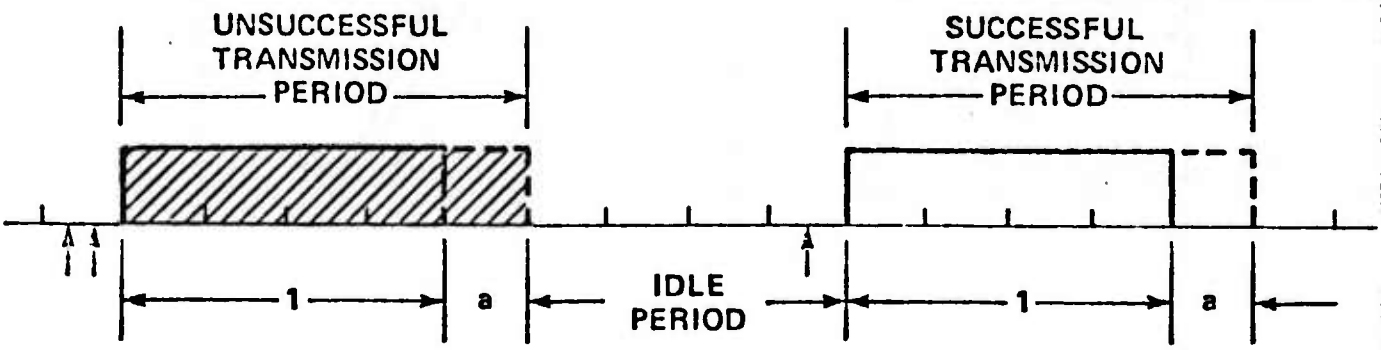


Figure 1 Slotted Nonpersistent CSMA: Transmission and Idle Periods.

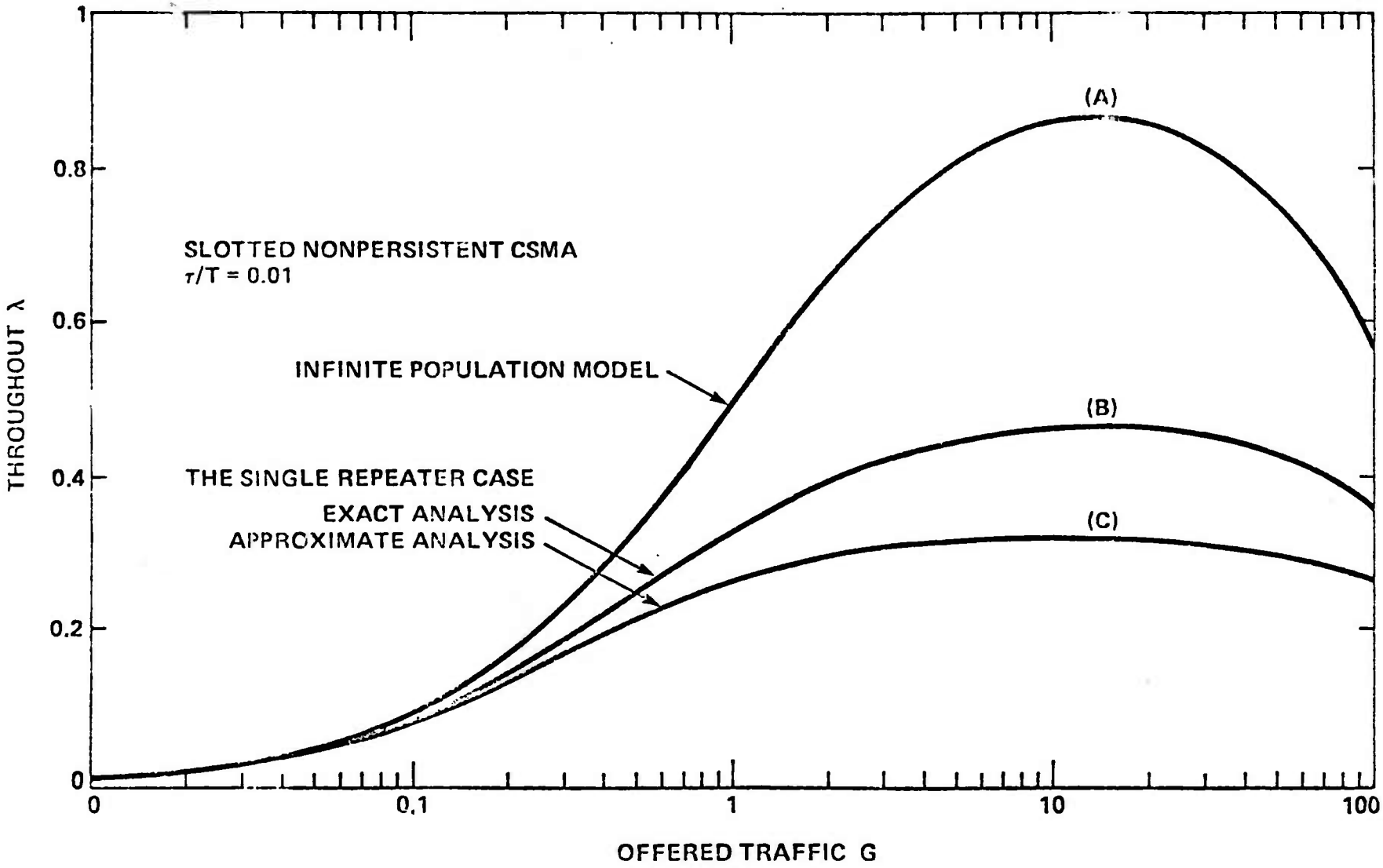


Figure 2 Nonpersistent CSMA: Throughput versus Offered Channel Traffic.

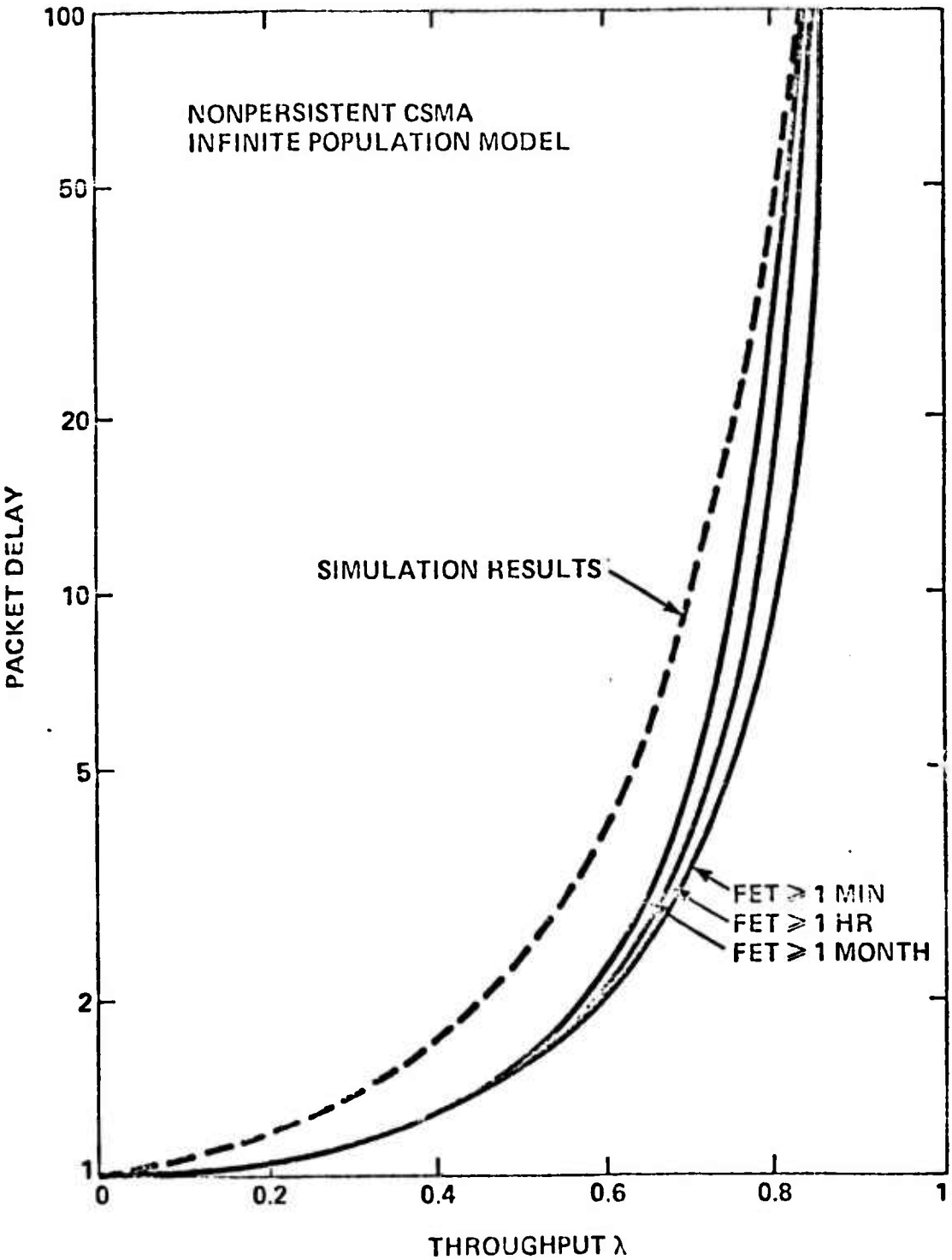


Figure 3 Nonpersistent CSMA: Throughput-Delay Tradeoff for the Infinite Population (a = 0.01).

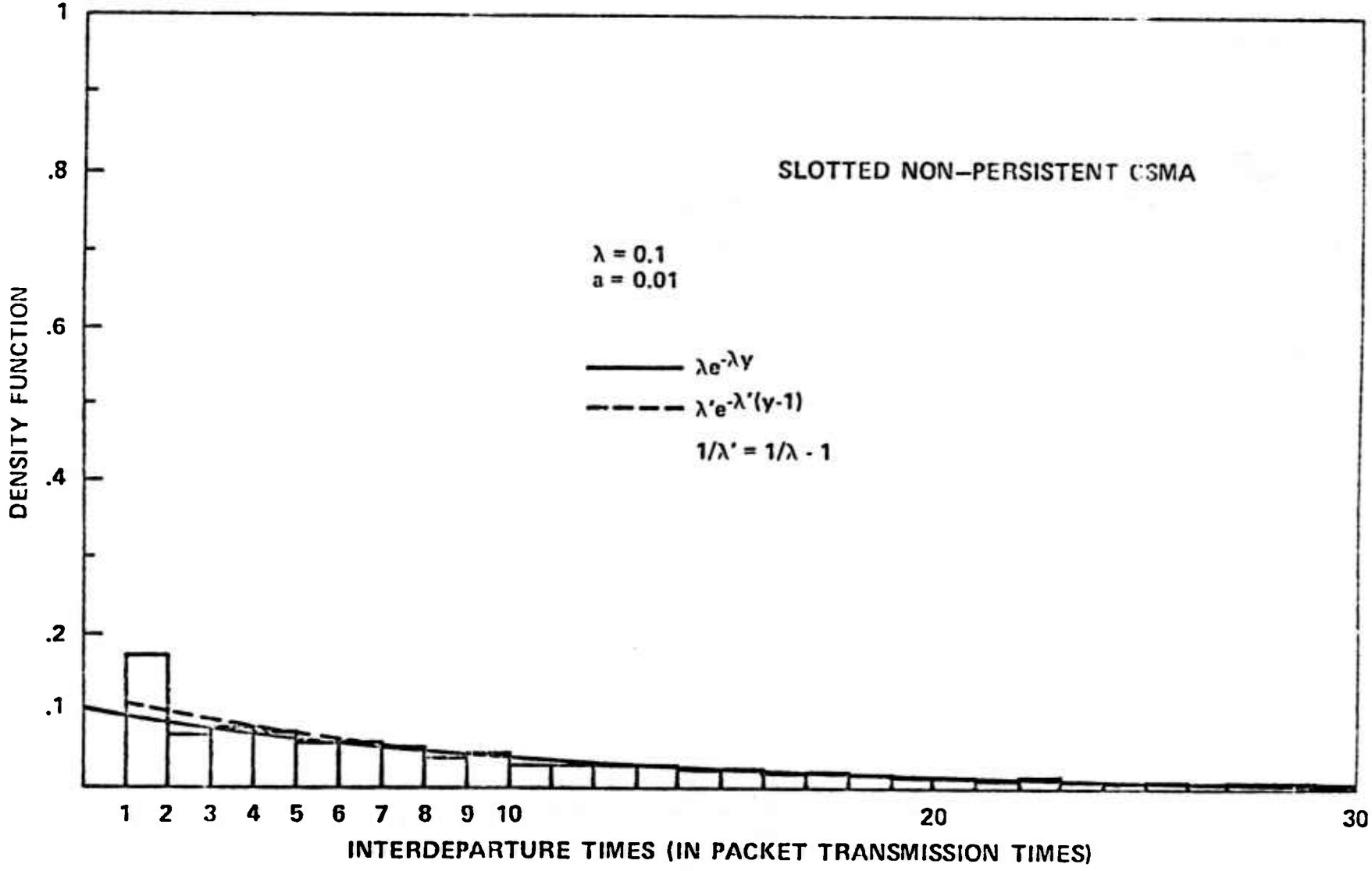


Figure 7-6(a) Histograms of Interdeparture Times in Slotted Non-Persistent CSMA (S_r=0.1).

Figure 4(a) Histograms of Interdeparture Times in Slotted Nonpersistent CSMA (λ = 0.1).

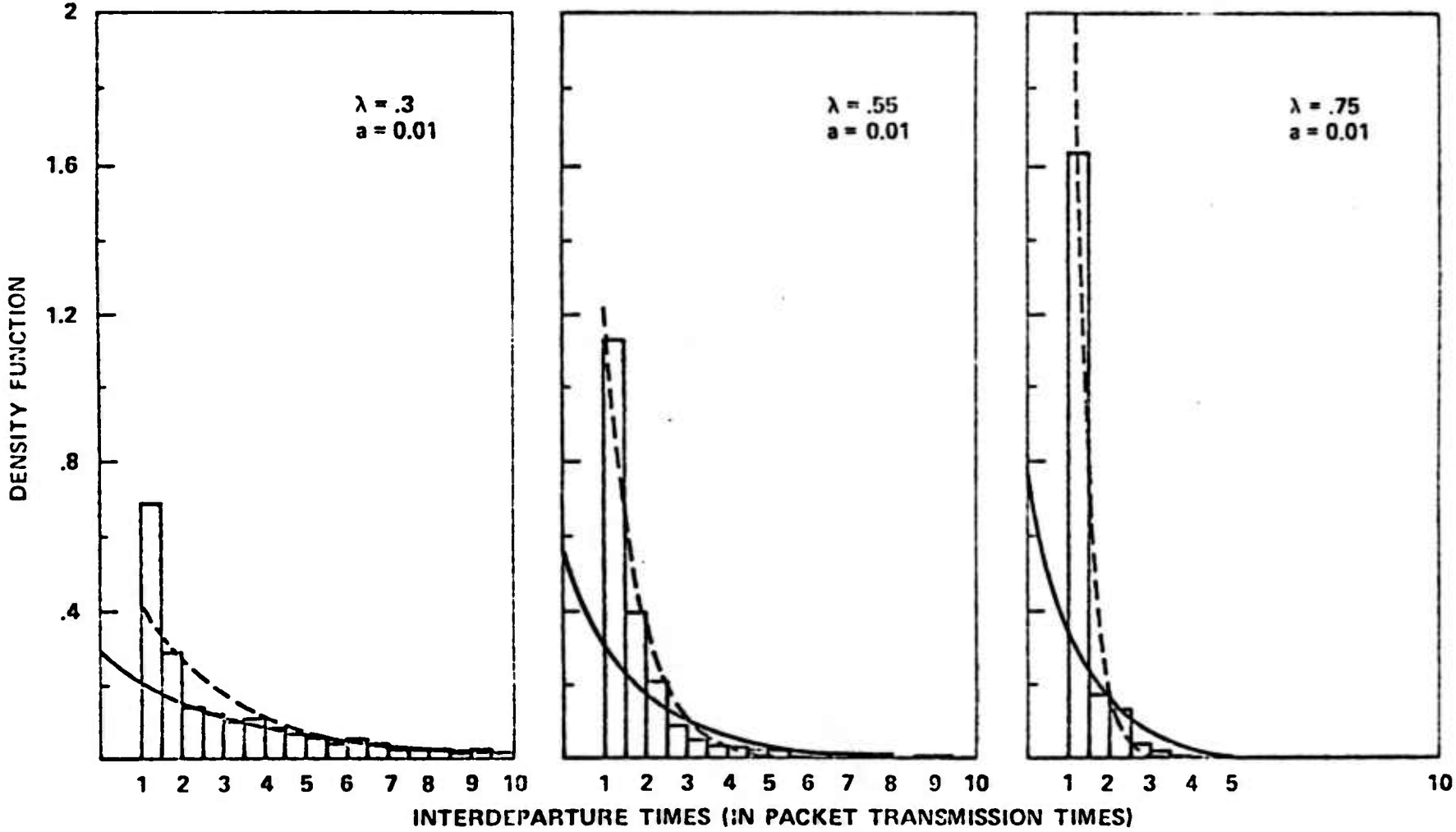


Figure 4(b) Histograms of Interdeparture Times in Slotted Nonpersistent CSMA (λ = 0.3, 0.55 and 0.75).

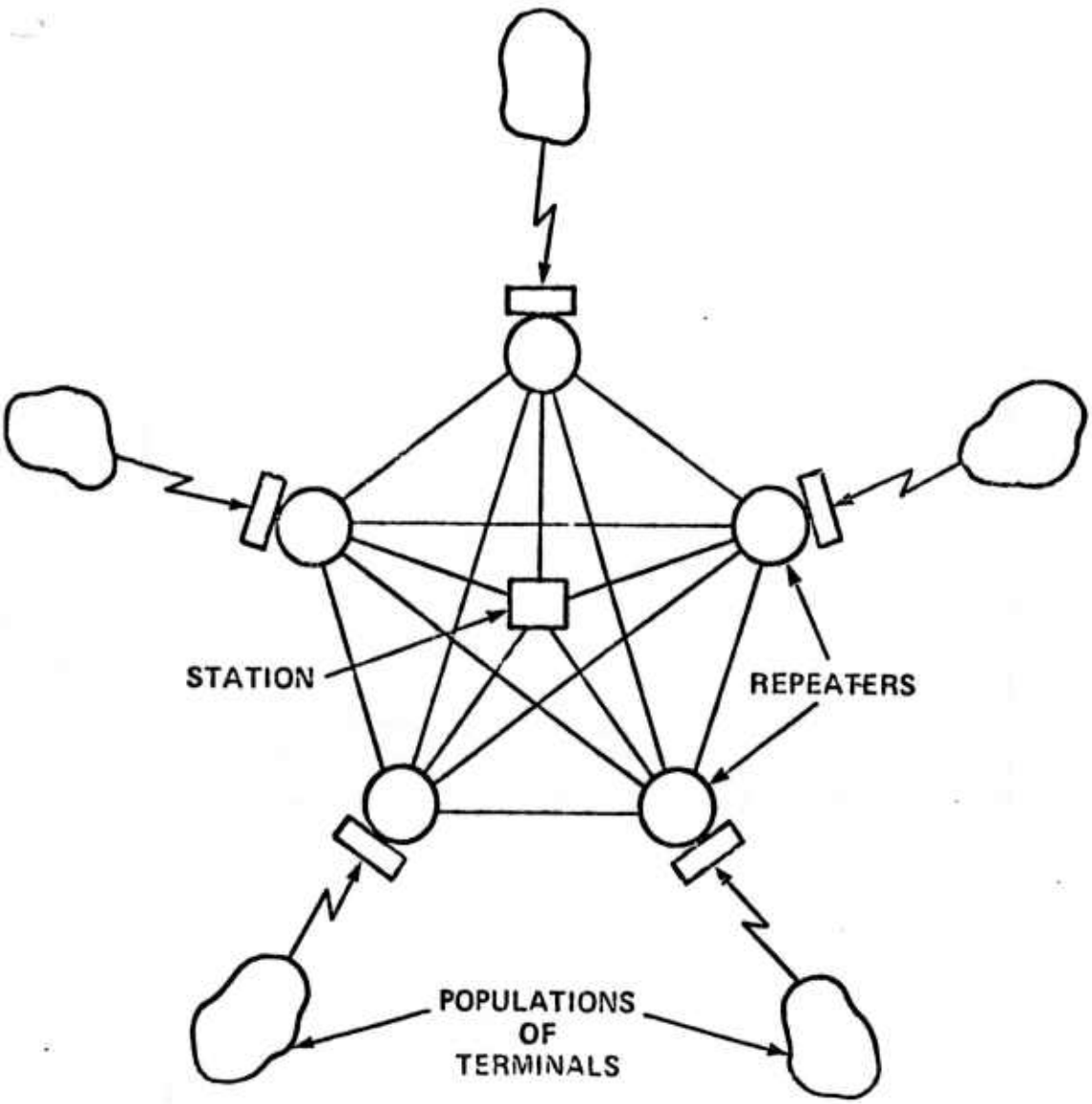


Figure 5 A Two-Hop Fully-Connected Configuration.

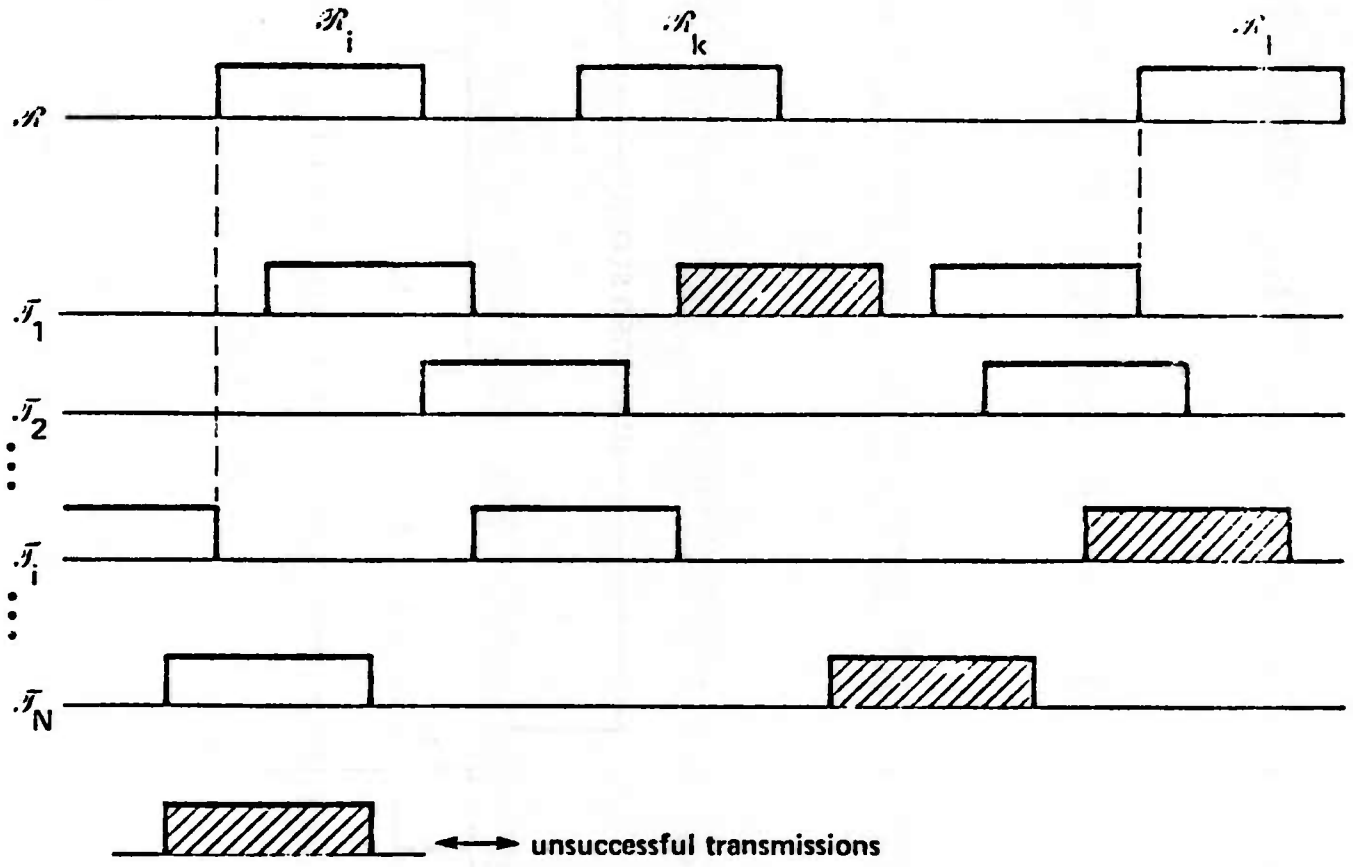


Figure 6 R and T_i Time Lines Exhibiting Packet Transmissions.

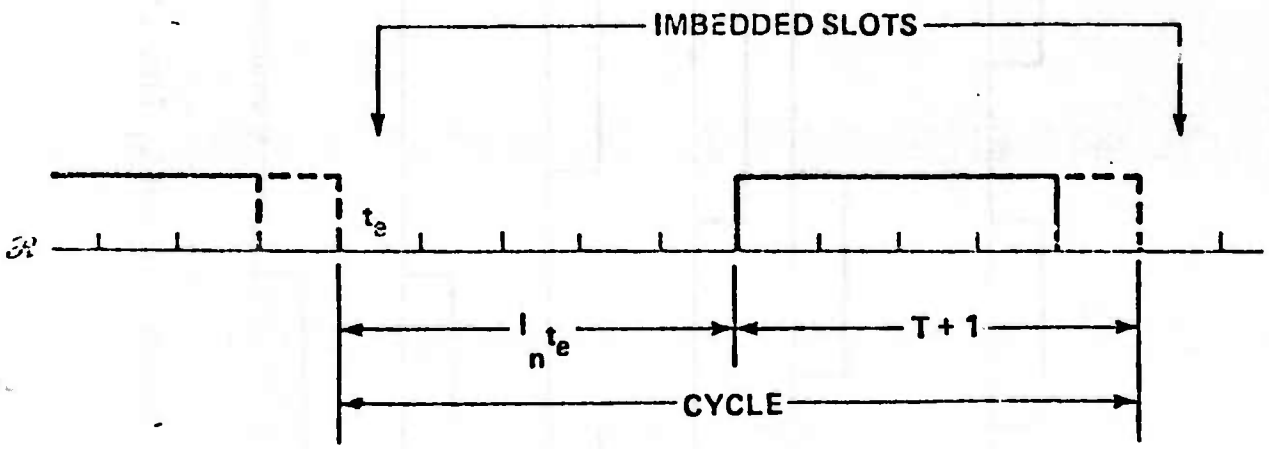
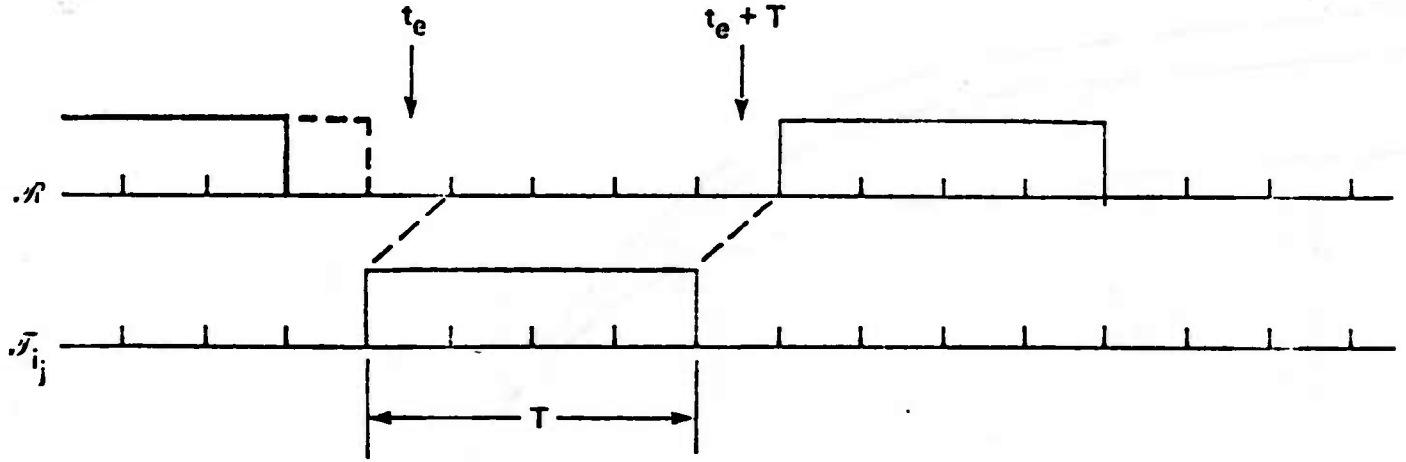
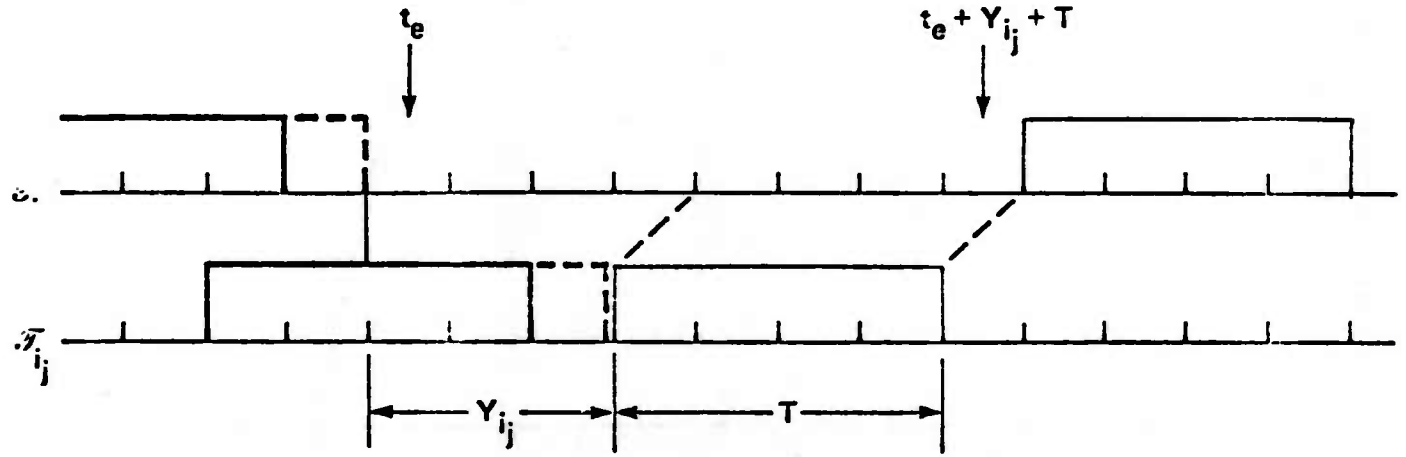


Figure 7 The Imbedded Slots on Time Line R.



(a) end of cycle occurs in idle period



(b) end of cycle occurs in transmission period

Figure 8 Determination of Minimum Idle Time for Possible Completion of Packet Reception.

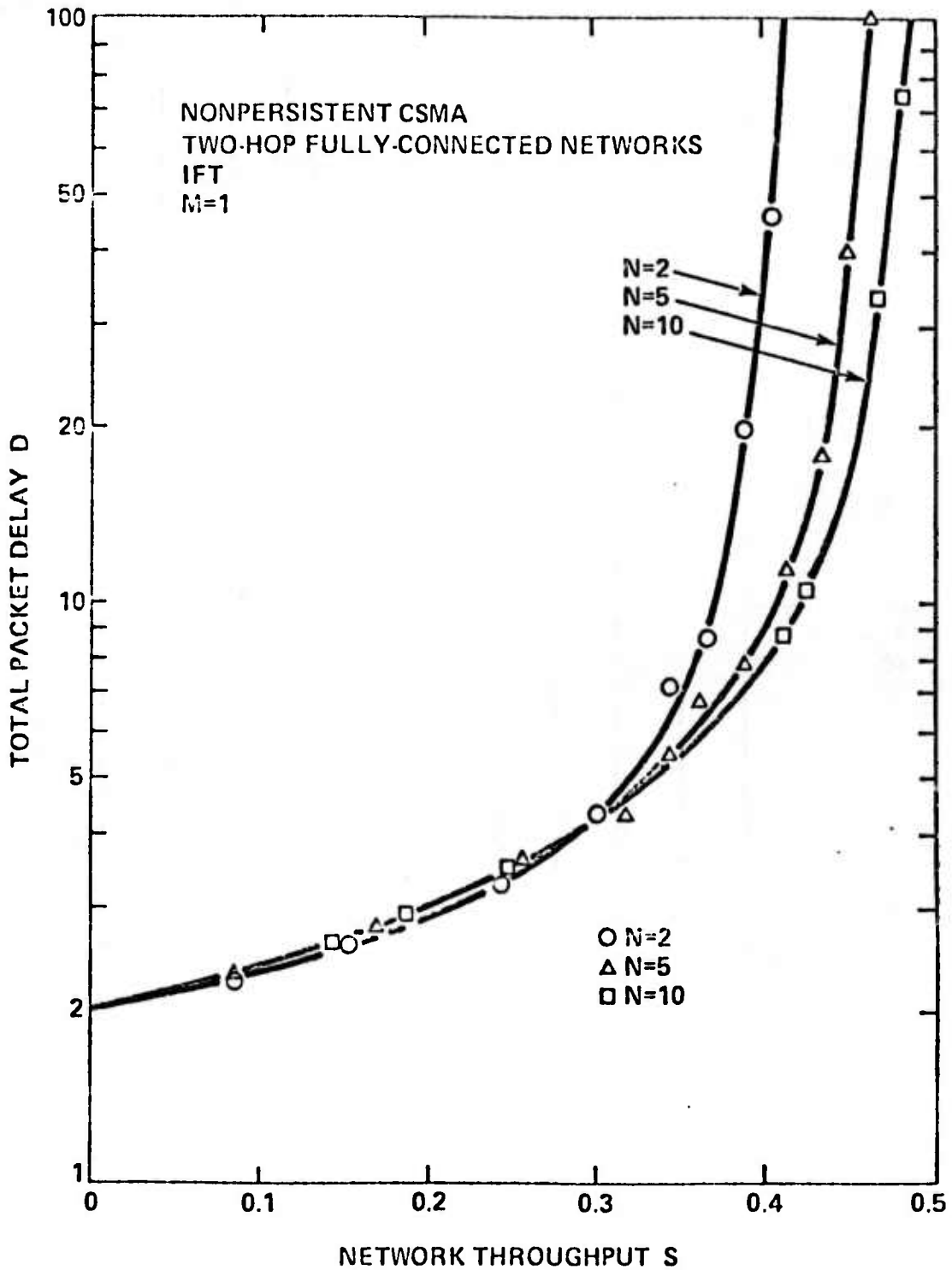


Figure 9 Throughput-Delay Tradeoffs in Nonpersistent CSMA Two-Hop Fully-Connected Networks ($a = 0.01$).

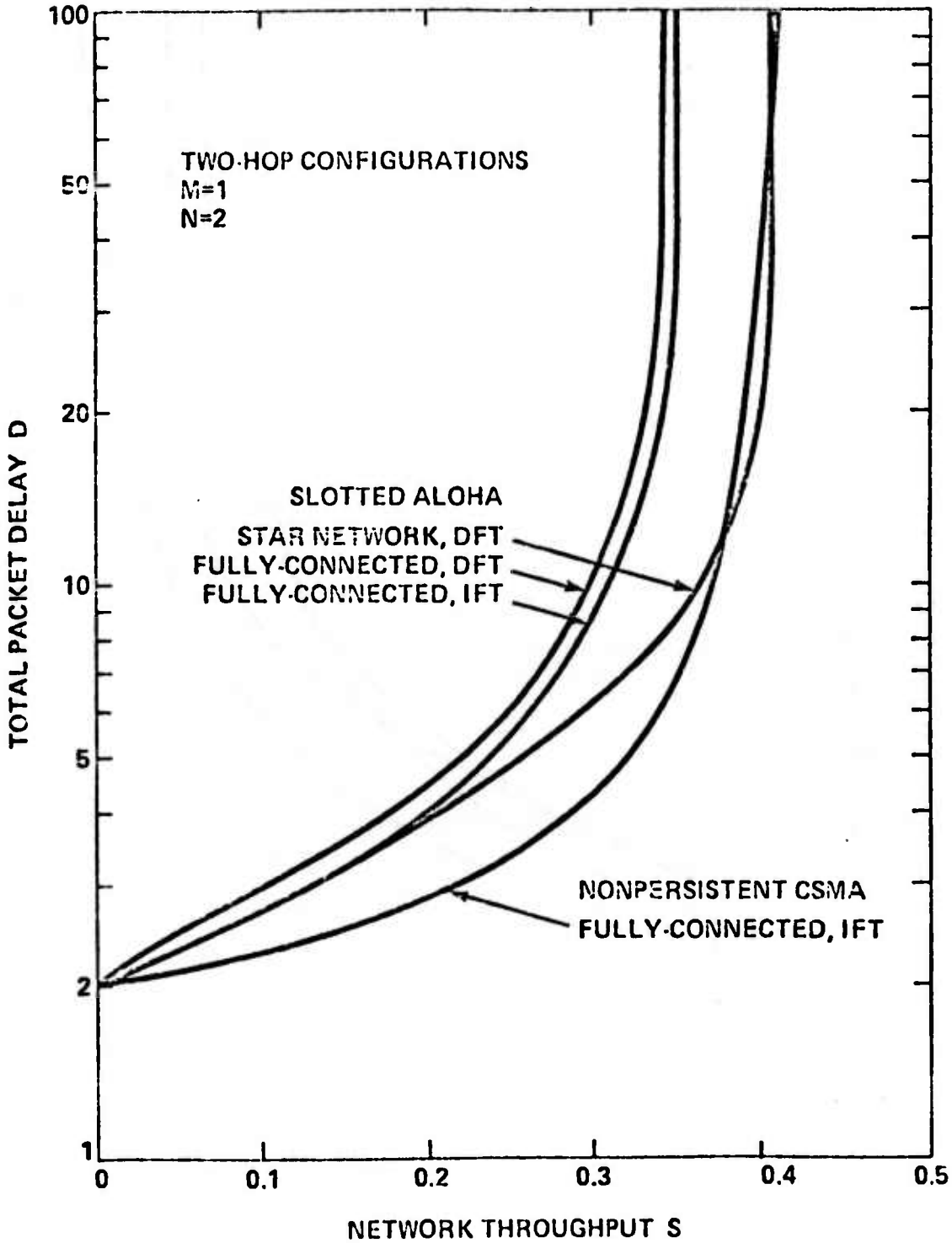


Figure 10 Comparison between Slotted ALOHA and Nonpersistent CSMA (a = 0.01) for N = 2.

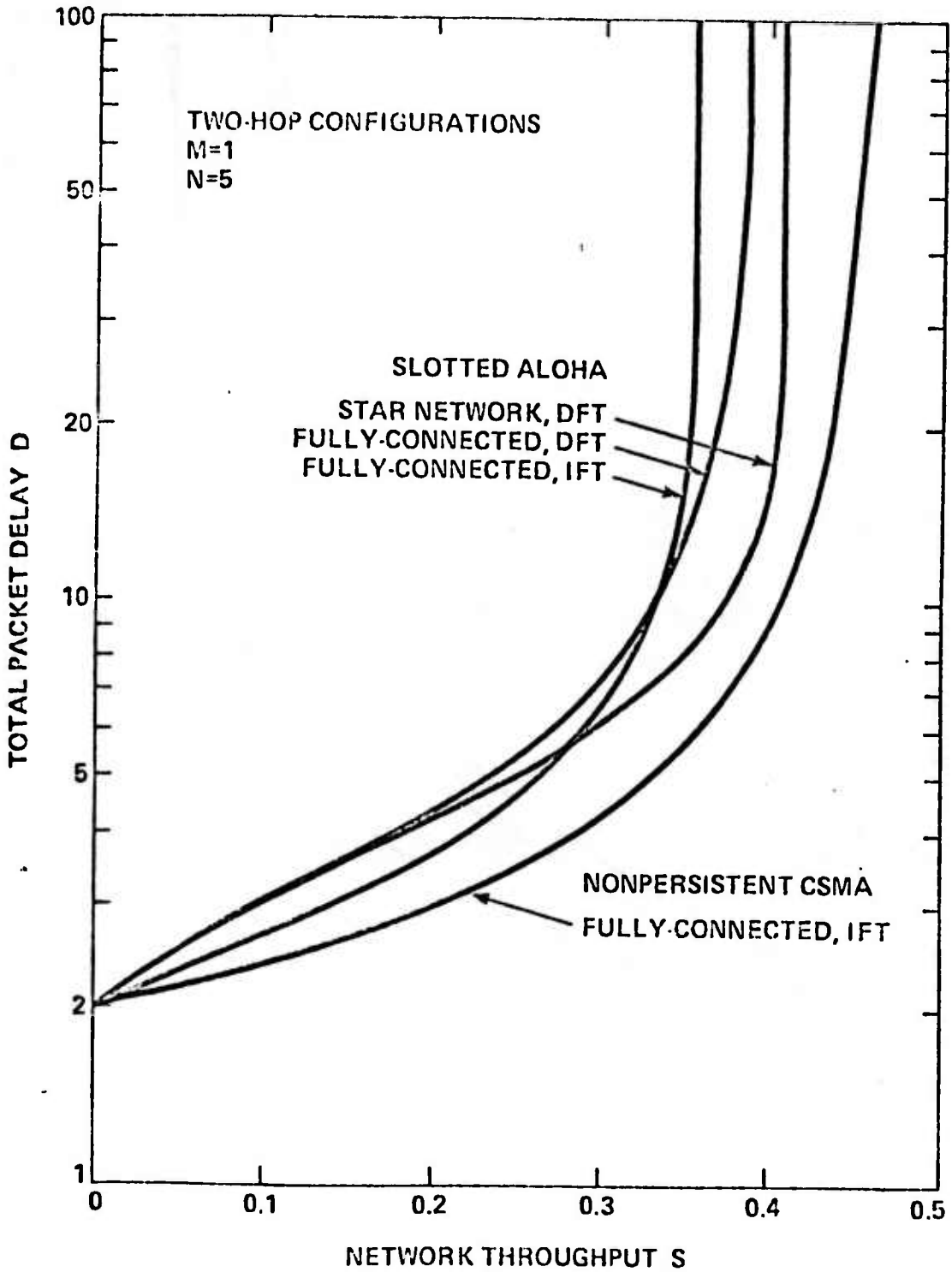


Figure 11 Comparison between Slotted ALOHA and Nonpersistent CSMA (a = 0.01) for N = 5.

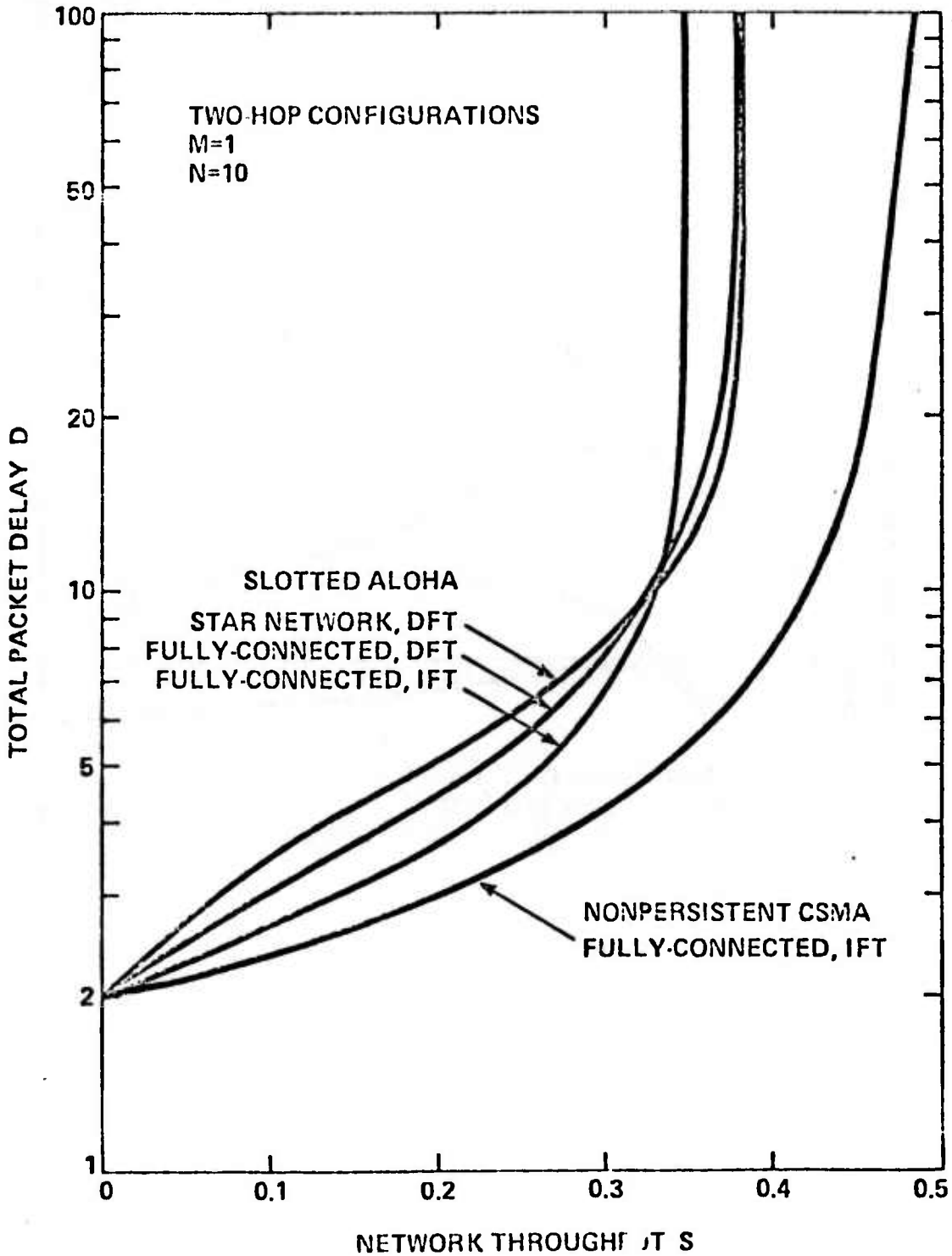


Figure 12 Comparison between Slotted ALOHA and Nonpersistent CSMA (a = 0.01) for N = 10.

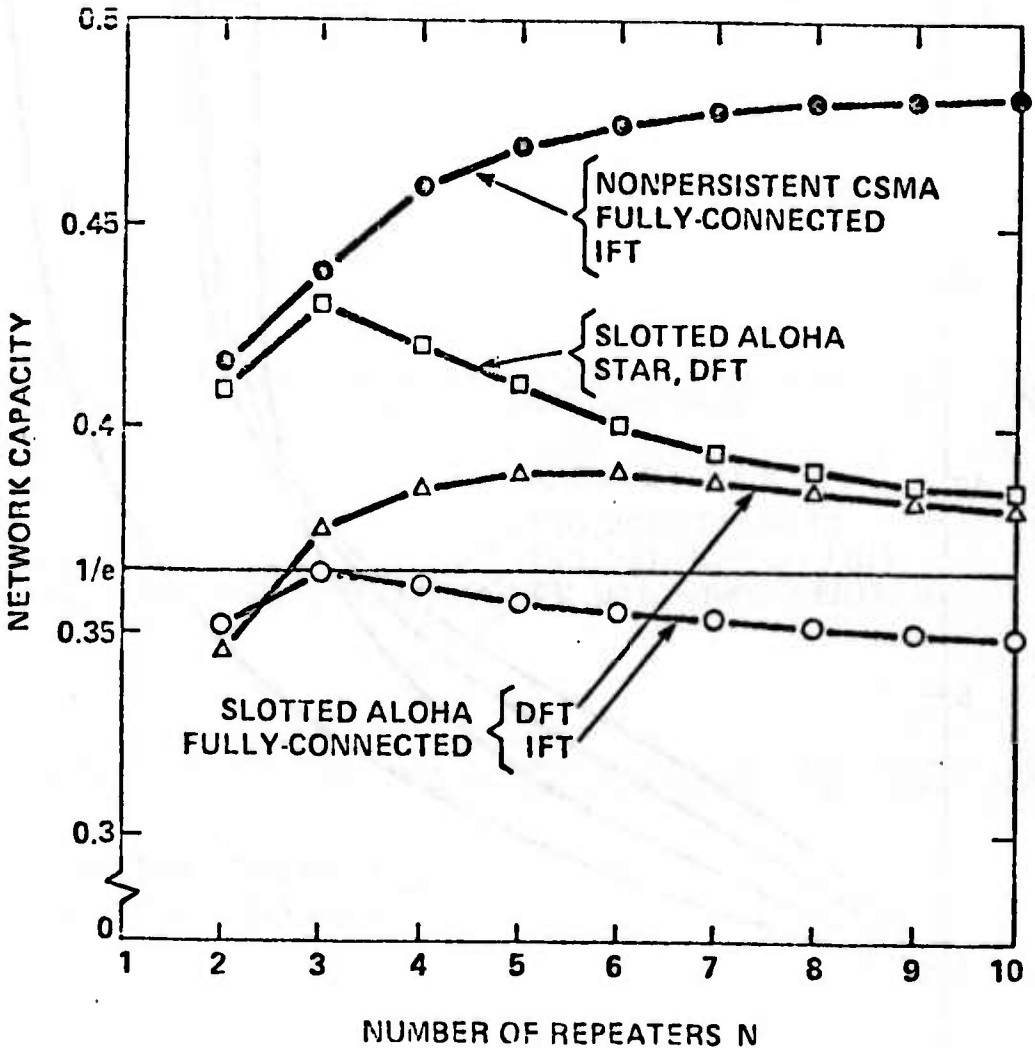


Figure 13 Network Capacity versus N for Slotted ALOHA and Nonpersistent CSMA Netowrks (a = 0.01).